A Semi-Automatic 3-Port Network Analyzer

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***Abstract*—A semi-automatic three-port network analyzer has been developed. The three-port network analyzer consists of an automatic two-port network analyzer, a switching network for RF rerouting and a software package. The design of the three-port network analyzer is based on a new method for obtaining accurate three-port S-parameters from measured two-port parameters. This method is based on a flow-chart description of the measurements. The resulting, overdetermined, nonlinear, equation system is solved numerically. No assumptions are made about the device under test or the terminations apart from that the latter must be known. The efficiency of the method is demonstrated by characterization of a two-way Wilkinson power divider. The three-port network analyzer has been used to characterize dual-gate MESFET's.**

I. Introduction

ACCURATE S-parameter measurements of multiport devices such as the dual-gate MESFET is nontrivial as commercially available network analyzers such as the HP-8510 and Wiltron W-360 are designed for measuring one- and two-ports.

Devices with more than two ports are usually characterized by a number of two-port measurements carried out with the ports not connected to the network analyzer terminated in low-reflection loads. The reflections from these terminations introduces errors. These errors can usually be neglected if the device has standard connectors and the frequency is moderate. At high microwave frequencies or when fixtures must be used, the nonreflective termination assumption is not valid and the measured two-port data must be renormalized as well as concatenated to result in accurate multiport data.

Solutions to this concatenation and renormalization problem have been presented by Woods [1] and Tippet and Speciale [2]. The terminating impedances must be known in both cases. Dropkin [3] has simplified the transform derived by Tippet and Speciale. He has also pointed out that the matrix inverse needed does not always exist. A simpler solution for three-port devices has been described by Goldberg, *et al.* [4]. The mathematical simplification has been achieved by terminating each port in two different impedances and measuring the two-port parameters for each case. The two impedances should be well separated in the Smith-chart to avoid numerical problems. For active devices, this might be a problem as oscillations must be avoided. Rautio [5] has also studied the problem of measuring an imperfectly terminated three-port with a two-port network analyzer. He gives two solutions: an iterative solution based on a recursive algorithm and a solution based on closed form equations. The recursive algorithm only converges under certain conditions and does not use all measured data. The closed form equations, although different from those discussed above, are complex and require matrix inversions.

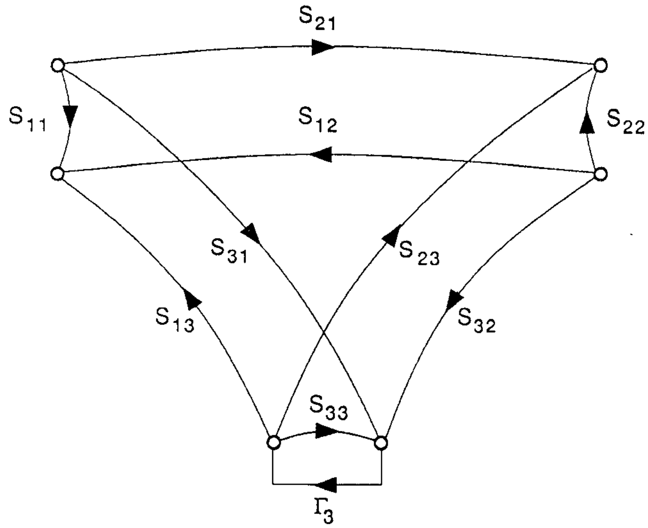

Fig. 1. Flow-chart of two-port measurement of three-port device.

Speciale [6] has described the theory of a multiport network analyzer based on the Super-TSD calibration procedure [7]. Lin and Ruan [8] have described a method for measuring n-ports ($n \geq 3$) with a three-port network analyzer. Their approach depends on the use of ideal isolators.

Section II of this paper describes a solution to the problem of measuring a three-port device with a two-port network analyzer. The solution is based on a flow-chart description of the measurement situation. The impedance renormalization and the conversion from two-port S-parameters into three-port S-parameters is solved as a nonlinear least squares problem. Measurement results that verify the method are reported in Section III. Section IV describes a semi-automatic three-port network analyzer whose design is based on the method described in Section II.

II. Theory

The flow-chart of Fig. 1 describes the measurement of a three-port device by using a two-port network analyzer. This 'measurement' results in the following two-port S-parameters:

$$\begin{bmatrix} S_{11} + \dfrac{S_{13}S_{31}\Gamma_3}{1 - S_{33}\Gamma_3} & S_{12} + \dfrac{S_{13}S_{32}\Gamma}{1 - S_{33}\Gamma_3} \\ S_{21} + \dfrac{S_{23}S_{31}\Gamma_3}{1 - S_{33}\Gamma_3} & S_{22} + \dfrac{S_{23}S_{32}\Gamma}{1 - S_{33}\Gamma_3} \end{bmatrix}. \quad (1)$$

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This can be generalized to:

$$S_{ij}^{(m)} = S_{ij} + \frac{S_{ik}S_{kj}\Gamma_k}{1 - S_{kk}\Gamma_k} \qquad i,j,k = 1,2,3 \quad i \neq k, \quad j \neq k \tag{2}$$

where $S_{ij}^{(m)}$ is the measured parameter,

$S_{ij}, S_{ik}, S_{kj}, S_{kk}$ are parameters of the three-port and Γ_k is the reflection coefficient applied to port k.

Three two-port measurements are needed to cover all parameters of the three-port (note that the reflection coefficients of the three-port are measured twice: $S_{ii}^{(m)}$ and $S_{ii}^{(m2)}$). This results in twelve versions of (2) and we are only interested in determining the nine complex parameters of the three-port This indicates that we may not need to know Γ_1, Γ_2, and Γ_3. However, tests using real and simulated measurement data have shown that the resulting nonlinear equation system has several solutions and is numerically ill behaved.

If, on the other hand, Γ_1, Γ_2, and Γ_3 are known we may generate a nonlinear least squares problem:

$$\min_{x \in \Re^{18}} \frac{1}{2} \sum_{i=1}^{24} f_i(x)^2 \tag{3}$$

where

$$x = \{\mathrm{Re}(S_{11}), \mathrm{Im}(S_{11}), \mathrm{Re}(S_{12}), \cdots, \mathrm{Im}(S_{33})\}^T \tag{4}$$

and

$$\begin{aligned} f_1 &= \mathrm{Re}\left\{ S_{11}^{(m)} - S_{11} - \frac{S_{13}S_{31}\Gamma_3}{1 - S_{33}\Gamma_3} \right\} \\ f_2 &= \mathrm{Im}\left\{ S_{11}^{(m)} - S_{11} - \frac{S_{13}S_{31}\Gamma_3}{1 - S_{33}\Gamma_3} \right\} \\ &\vdots \\ f_{24} &= \mathrm{Im}\left\{ S_{33}^{(m2)} - S_{33} - \frac{S_{13}S_{31}\Gamma_1}{1 - S_{11}\Gamma_1} \right\}. \end{aligned} \tag{5}$$

Hence, the nonlinear least squares problem is treated as a nonlinear minimization problem which is solved numerically. Proven algorithms for solving the nonlinear least squares problems are available for free (from NETLIB for instance) or as part of commercial software (NAG, IMSL, Mathematica, etc.). A modified Levenberg–Marquardt algorithm [9] in the form of the IMSL-routine DBCLSF [10] was used in this case. This algorithm searches for the minimum of (3) and the search direction is governed by the Jacobian J which is a 24×18 matrix where the elements are the partial derivatives of the function F

$$F = \{f_1, f_2, \cdots, f_{24}\}^T \tag{6}$$

$$J_{ij} = \frac{\partial f_i}{\partial x_j}. \tag{7}$$

The Jacobian is numerically approximated in DBCLSF. Starting values for the S-parameters are obtained by temporarily assuming that all $\Gamma_k = 0$. The variation of the variables are restricted throughout the optimization to *starting value* ± 0.5.

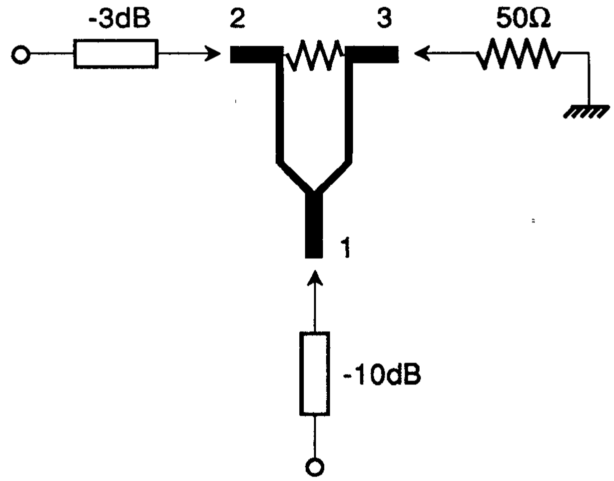


Fig. 2. The Wilkinson power divider and the terminations used in the verification measurement.

The method described above was tested using simulated two-port 'measurement' data based on typical Dual-Gate MESFET parameters and different terminations. These tests showed that the program converged to a solution with three correct decimals for the three-port S-parameters in typically less than five iterations. Perturbation of the input data results in output data deviations of the same magnitude as the perturbation.

III. Verification

A two-way Wilkinson power-divider with SMA-connectors was characterized to verify the procedure described in the previous section. The results from two characterizations are compared. The terminations and their port-allocations for the first case are shown in Fig. 2. (When measuring between Port 1 and Port 2, Port 3 was terminated in a 50 Ω load, etc.) The attenuators are coaxial attenuators from Narda and Hewlett Packard. The 50 Ω load is the precision load (HP 85052-60010) used to calibrate the network analyzer. The unused ports of the attenuators were left open ($|\Gamma_1| \approx 0.1, |\Gamma_2| \approx 0.5$ and $|\Gamma_3| \equiv 0$). The 50 Ω load was used for all three measurements in the second case. Fig. 3 shows the difference between terminating Port 2 in an 50 Ω load and in an open-ended 3 dB attenuator when measuring between Port 1 and Port 3. Fig. 4 shows both the result of processing data based on measurements using the terminations in Fig. 2 and data from the second ('ideal') case. $S_{33}, S_{12}, S_{13}, S_{31}$, and S_{32} coincide with the data shown and are left out for clarity.

IV. System Description

The three-port network analyzer system consists of four parts (see also Fig. 5):

- A standard two-port network analyzer capable of TRL calibration.
- A three-port fixture.
- A RF- and DC-rerouter system that is the interface between the three-port fixture and the two-port network analyzer.

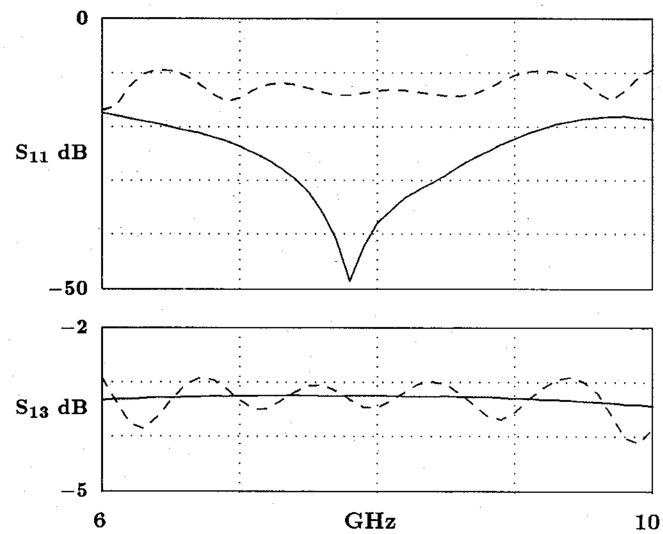


Fig. 3. Comparison of two-port measurements of a Wilkinson power divider with Port 2 terminated in a precision 50Ω load (solid line) and in an openended 3dB attenuator (broken line).

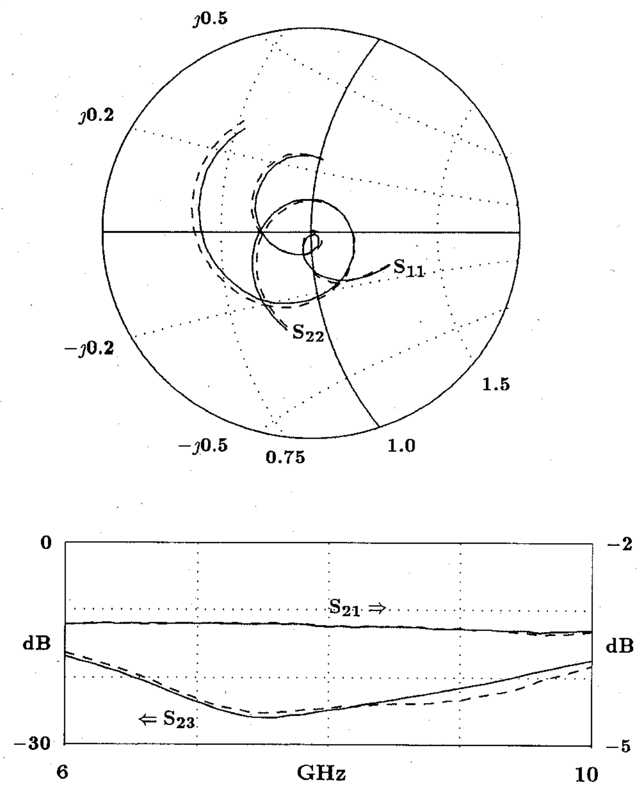


Fig. 4. Processed three-port data of a Wilkinson hybrid measured with imperfect terminations (broken line) compared with data from an 'ideal' measurement (solid line).

- A computer program for calculating three-port S-parameters from measured data.

A bias supply is added when active devices are measured.

A. 3-Port Fixture

The fixture must meet the following requirements:

- It must interface the transistor chip with the coaxial ports of the network analyzer.
- It should be designed for some in-fixture calibration like the TRL calibration.
- It must be possible to measure the third-port termination accurately.

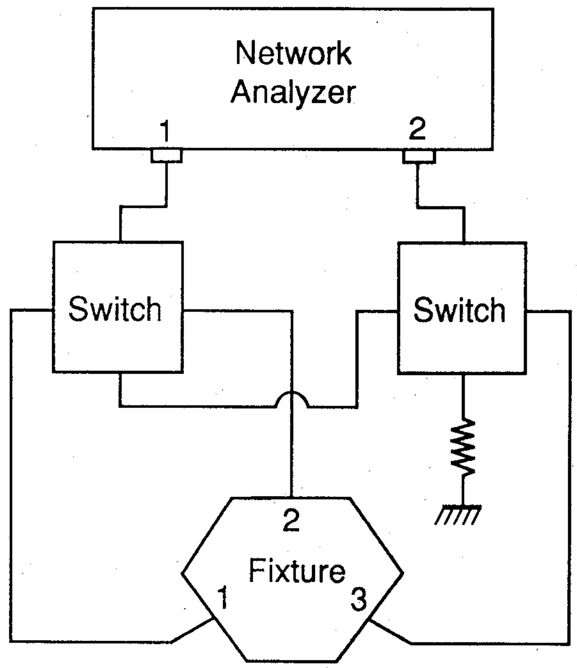


Fig. 5. Dual-Gate FET RF measurement system. The bias circuitry is left out for clarity.

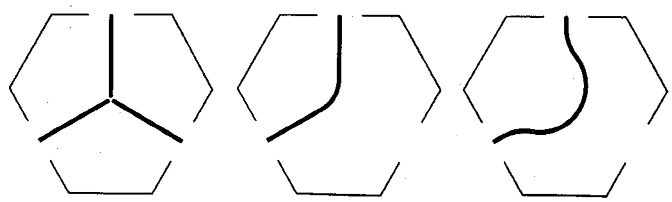


Fig. 6. 3-port fixture layout: transistor, 'Thru', and 'Line' layouts.

A symmetrical, hexagonal geometry, Fig. 6, meets these requirements. The size of the fixture is dictated by the need for a 7.1 mm ($\approx \lambda/4$ @ 5 GHz) difference between 'Line' and 'Thru'. This length-difference makes accurate calibration over the 1–9 GHz band possible. Several shorter 'Lines' can be added for wide-band calibration [11]. The radii of the bends used are not expected to significantly decrease the quality of the TRL-calibration [12]. The 'Reflect' standard is a transistor carrier without a transistor. The three-way symmetry of the fixture makes it possible to use the same standards between all three port-pairs.

A 0.635 mm thick RT/Duroid 6006 substrate ($\epsilon_r = 6.0$) is used for the fixture. A 50 Ω line on this substrate is calculated (Linecalc™) to be 0.93 mm wide. This width fits the center conductor of the coaxial connector well without being too wide compared to the transistor chip. A short taper is introduced close to the chip to reduce coupling between the measurement ports, Fig. 7.

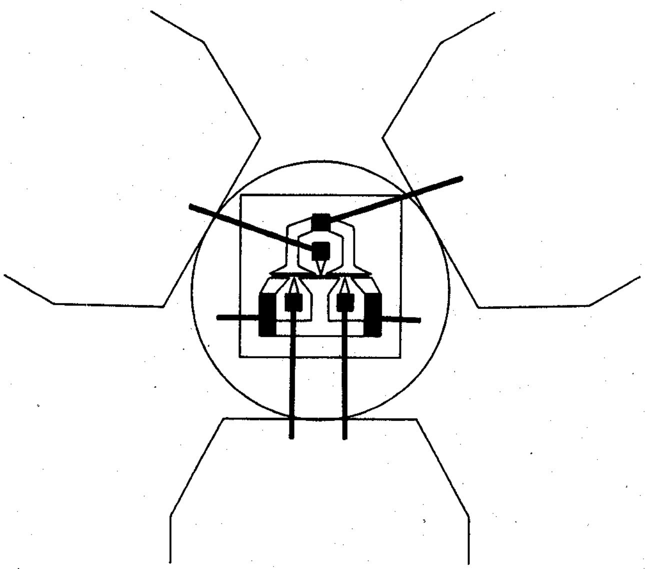


Fig. 7. Close-up of transistor in fixture.

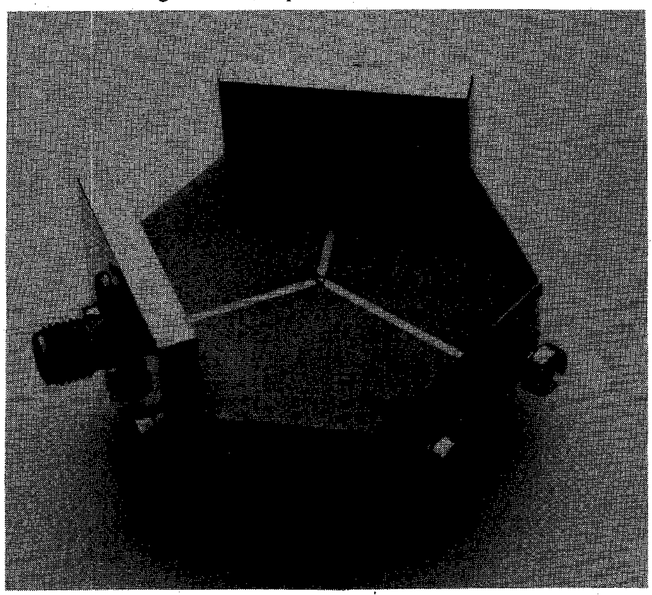


Fig. 8. Complete fixture with one port removed.

Each calibration standard is bonded with silver-epoxy glue to a brass carrier to which the coaxial connector mounts are attached. The coax-to-microstrip transitions are part of the error-networks and it is here and not in the SMA interface the signal-path is disconnected between the different calibration and measurement steps. Hence, the coax-to-microstrip transitions must be highly reproducible. The connector mounts are designed with a lip that rests on the substrate when the center conductor rests firmly on the microstrip line. The center conductor tip is designed for nonsoldering connection (Omni Spectra OSM 2052-5277-02).

Nonambiguous, repeatable connection between the connector mount and the microstrip ground plane is essential. This is enforced by cutting a groove in the side of the carrier so that contact is made between the mount and the carrier as close to the substrate as possible. Fig. 8 shows a photo of the fixture.

The fixture was studied in the time domain using a HP-8720 network analyzer. The reflection at the coax-microstrip transition was less than $0.7 \cdot 10^{-3}$ and the impedance of the 'Line'-line was within $\pm 0.5\,\Omega$ of the 50 Ω calibration load.

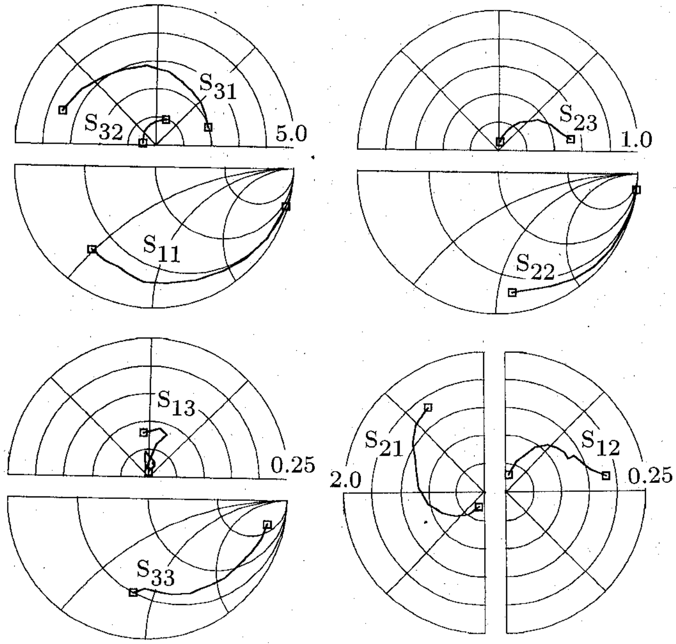


Fig. 9. The measured S-parameters for 1–10 GHz of NE25000 at $V_{DS} = 3.00\,\text{V}, V_{G1S} = -1.22\,\text{V}, V_{G2S} = -0.19\,\text{V}, I_{DS} = 9.7$ mA.

B. RF- and DC-Rerouting System

The measurement-port switching is carried out by two transfer switches (Sivers PM 7551). These switches are highly reproducible [13]. The two switches, a bias-T (for the transistor terminal not connected to the network analyzer), and support circuitry (logic circuits and drivers for the switches) are placed in a box that is connected to the network analyzer, the bias controller and to a DC-supply. The three-port fixture is placed on top of this box. The support circuitry enables the user to reroute the RF by simply turning a knob. The bias voltages automatically stay at the correct transistor terminals. Three out of four possible settings of the two switches are used:

a) ANA Port 1 to fixture Port 1 via the left switch. Ana Port 2 to fixture Port 2 via both switches. Fixture Port 3 to load via the right switch.
b) Left switch unchanged. Right switch changed: ANA Port 2 to fixture Port 3 and fixture Port 2 to load.
c) Right switch as in 2. Left switch changed: ANA Port 1 to fixture Port 2 and fixture Port 1 to load via both switches.

C. Calibration and Measurement Sequence

1) Calibration:

a) Calibrate between ports 1 and 2 (switch-setting 1). Save the calibration.
b) Connect 'Thru' between ports 1 and 3. Measure and save Γ_3.
c) Repeat a) and b) for port-pairs 1–3 and 2–3 (switch-settings 2 and 3, respectively).

2) Device measurement:

a) Insert a carrier containing a device and bias.

b) Measure between ports 1 and 2 using the corresponding calibration (switch-setting 1). Save corrected two-port S-parameters.

c) Repeat c) for port-pairs 1–3 and 2–3 (switch-settings 2 and 3, respectively).

3) Compute three-port S-parameters from measured two-port S-parameters and termination reflection coefficients:

The system may quite easily be converted into a fully automatic three-port network analyzer by adding a computer interface to the RF-rerouting switch-system. An external computer could then control the switches, the two-port network analyzer, and calculate the three-port S -parameters.

D. Accuracy

No standards of metrology quality are available for the microstrip transmission medium. This has led to the practice of using TRL-type calibration techniques where only the precision of the impedance of the 'Line' standard and the reproducibility of the connection interfaces affects the calibration quality. Direct verification of the calibration quality is not possible but the reproducibility was checked after each measurement session. This was done by measuring the 'Thru' standard. The magnitudes of S_{11} and S_{22} were typically around -50 dB and only once exceeded -30 dB. The magnitudes of S_{12} and S_{21} never deviated more than 0.25 dB from the ideal response and the phase error never exceeded $1°$. The $|\Gamma_k|s$ were usually around 0.1 and were always below 0.2 so the three-port parameter calculations are expected to behave as well as in the simulations and the verification.

E. Measurement Example

Fig. 9 shows measured data for a dual-gate MESFET from NEC. Gate 1 = port 1, gate 2 = port 2 and drain = port 3.

V. Conclusions

The S-parameters of three-ports such as dual-gate FET's can be accurately measured by using the semi-automatic three-port network analyzer described in this paper. A flow-chart description of the measurement situation results in an over-determined, nonlinear equation system. A modified Levnberg–Marquardt algorithm is used to solve this equation system. The resulting algorithm is consistent and robust, this is demonstrated by the characterization of a two-way Wilkinson power-divider.

References

[1] D. Woods, "Multiport-network analysis by matrix normalization employing voltage-wave S-parameters with complex normalization," *Proc. IEE,* vol. 124, no. 3, pp. 198–204, Mar. 1977.

[2] J. C. Tippet and R. A. Speciale, "A rigorous technique for measuring the scattering matrix of a multiport device with a 2-port network analyzer." *IEEE Trans. MTT,* vol. 30, no. 5, pp. 661–666, May 1982.

[3] H. Dropkin, "Comments on 'A rigorous technique for measuring the scattering matrix of multiport devices with a two-port network analyzer'," *IEEE Trans. MTT,* vol. 31, no. 1, pp. 79–81, Jan. 1983.

[4] S. B. Goldberg, M. B. Steer, and P. D. Franzon, "Accurate experimental characterization of three-ports." *IEEE MTT-S Int. Microwave Symp. Digest,* Boston, MA, 1991, pp. 241–244.

[5] J. C. Rautio, "Techniques for correcting scattering parameter data of an imperfectly terminated multiport when measured with a two-port network analyzer," *IEEE Trans. MTT,* vol. 31, no. 5, pp. 407–412, May 1983.

[6] R. A. Speciale, "Multiport network analyzers: Meeting a design need," *Microwave System News,* pp. 67–89, June 1980.

[7] ——, "Generalization of the TSD network analyzer calibration procedure, covering n-port scattering-parameter measurements, affected by leakage errors," *IEEE Trans. MTT,* vol. 25, no. 12, pp. 1100–1115, Dec. 1977.

[8] W. Lin and C. Ruan, "Measurement and calibration of a universal six-port network analyzer," *IEEE Trans. MTT,* vol. 37, no. 4, pp. 734–742, Apr. 1989.

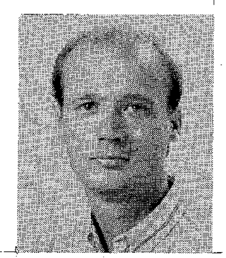
[9] P. Gill, W. Murray, and M. Wright, *Practical Optimization.* New York: Academic, 1981.

[10] IMSL User's Manual, Math/Library, version 1.0, pp. 879–881, Apr. 1987.

[11] R. B. Marks, "A multiline method of network analyzer calibration," *IEEE Trans. MTT,* vol. 39, no. 7, pp. 1205–1215, July 1991.

[12] S. Åkesson, "Utvärdering och kalibrering av mätfixtur." ("Measurement and evaluation of a measurement fixture.") Master's thesis, Division of Network Theory, Chalmers Univ. of Technology, Göteborg, Sweden, 1989, in Swedish.

[13] B. Albinsson, H. Guo, M. Schöön, and H.-O. Vickes, "A computer controlled noise parameter measurement system," Tech. Rep. TR9002, Division of Network Theory, Chalmers Univ. of Technology, Göteborg, Sweden, Aug. 1990.



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